Lecture #20

CS 170

Spring 2021

More NP-Complete Problems

- Review definitions of P, NP, etc
- All of NP → CSAT → SAT → 3SAT
- 3SAT → Independent Set (IS)

Vertex Cover (VC)   Clique

-

3SAT

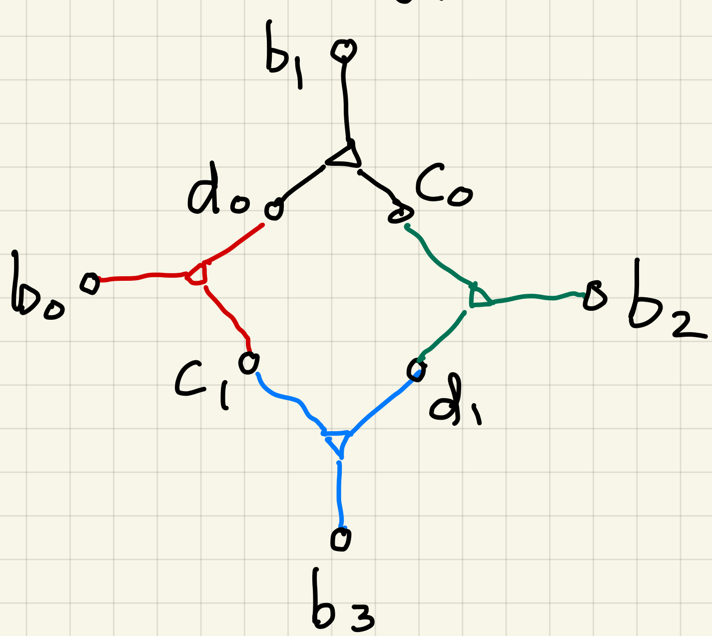
↓

Defining NP-hard and NP-complete

- P = "complexity class" of all relations R such that decide(R) costs poly$(|x|)$ (P = "polynomial")
- NP = all relations R such that given x, $\exists\, w$ of size $|w| = \text{poly}(|x|)$, so $V_R(x, w)$ costs poly$(|x|)$ when $R(x, w) = 1$ for some w
  - Ex: if $V_R(x, w)$ costs poly$(|x|)$
- Def: problem A is NP-hard if
- Def: problem A is NP-complete if

3SAT → 3D Matching (3DM) (1/3)

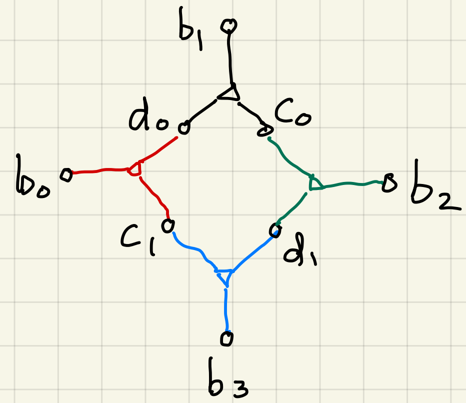
- 3DM: Given Sets $\{d_0, \dots, d_k\}$, $\{c_0, \dots, c_k\}$, $\{b_0, \dots, b_k\}$ and triples $\{(d_3, c_2, b_7), (d_1, c_3, b_2), \dots\}$: Is there a subset of triples where each d_i, c_i and b_i appears once?
- Need "gadgets" built from triples to model variables (T or F) and clauses $(x \vee \bar{y} \vee z)$
- Variable x: use 4 triples: $\Delta = (d_0, c_0, b_1)$, Δ, Δ, Δ



3SAT → 3D Matching (3DM) (2/3)

- For each clause, $(x \vee \bar{y} \vee z)$: add d_c and c_c

x:



var = T ⇒ △, △

var = F ⇒ △, △

$\bar{y}$:

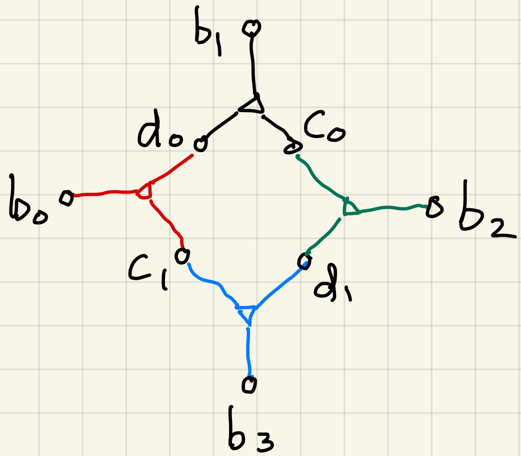
3SAT → 3D Matching (3DM) (3/3)

- What if each literal does not appear twice?

- What if some literal appears $<$ twice?

3D Matching (3DM) $\to$ Zero-one Equations (ZOE)

- ZOE: Solve (if possible) $Ax = 1$, each $A_{ij}, x_j \in \{0,1\}$
- A has
  -
  -
  -



$$A = \begin{array}{c} \\ d_0 \\ d_1 \\ c_0 \\ c_1 \\ b_0 \\ b_1 \\ b_2 \\ b_3 \end{array} \begin{array}{c} \begin{array}{cccc} \triangle & \triangle & \triangle & \triangle \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

- $(Ax)_i =$
- Ex: $\left(A \cdot \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}\right)_1 =$
- $Ax = 1$ iff

ZOE (Zero-One Equations) → ILP (Integer LP)

- ILP: need to find a "feasible" x : $Ax \le b$
-

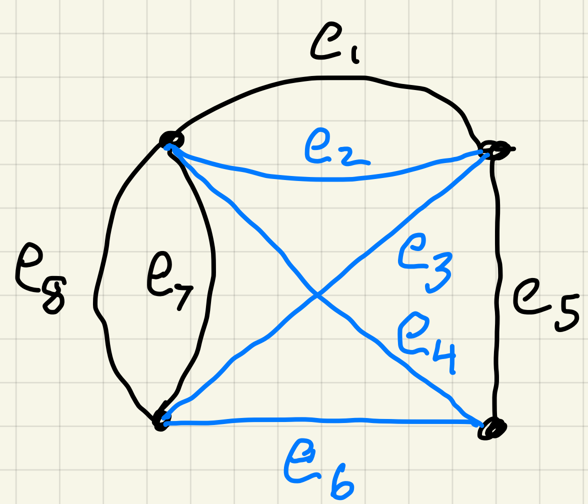
ZOE (Zero-One-Equations) (1/3)

$\longrightarrow$ RHC (Rudrata-Hamiltonian Cycle)

- RHC - find a cycle in G that visits each vertex once
- 2 Step Reduction:

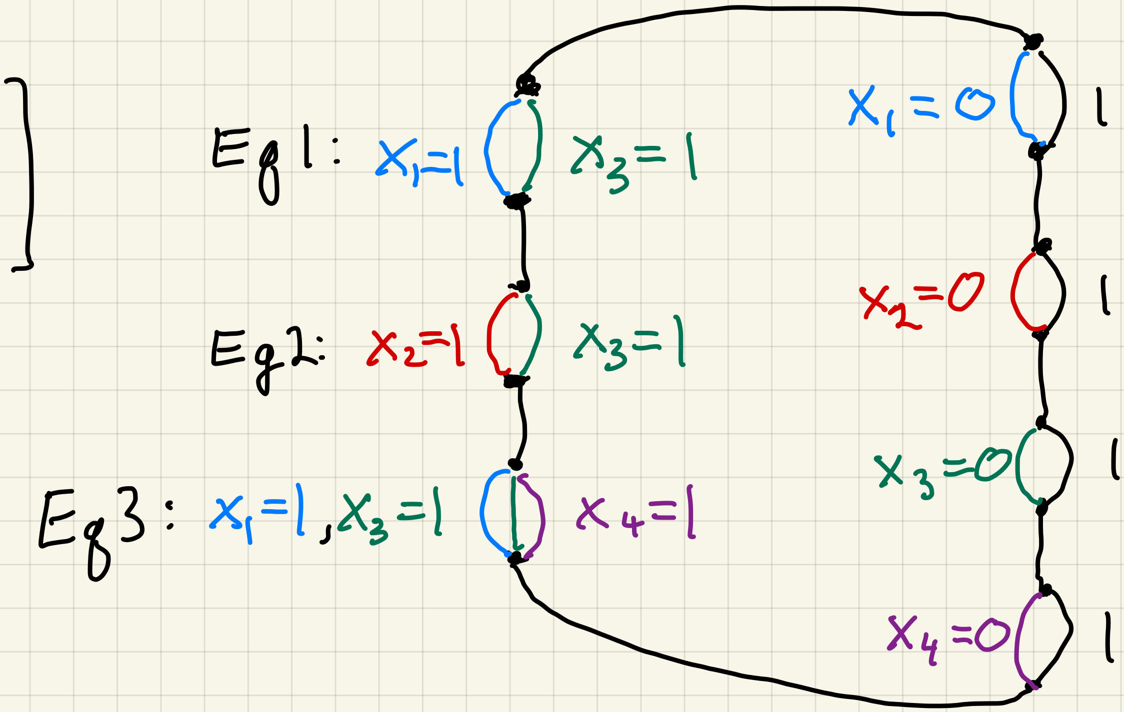
-

Ex: $C = \{(e_1, e_3), (e_5, e_6), (e_4, e_5), (e_3, e_7), (e_3, e_8)\}$



ZOE → RHC with paired edges (RHCwpe) (2/3)

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$



-

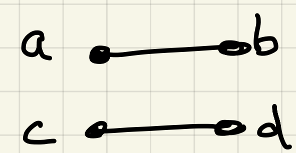
-

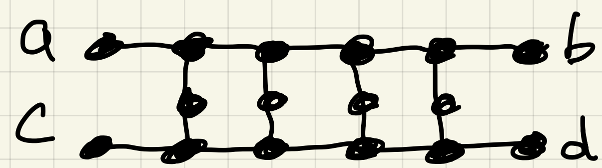
$$C = \{(\underset{\text{Eq1}}{x_1=1}, x_1=0), (\underset{\text{Eq 3}}{x_1=1}, x_1=0), (x_2=1, x_2=0), (\underset{\text{Eq1}}{x_3=1}, x_3=0), \ldots\}$$

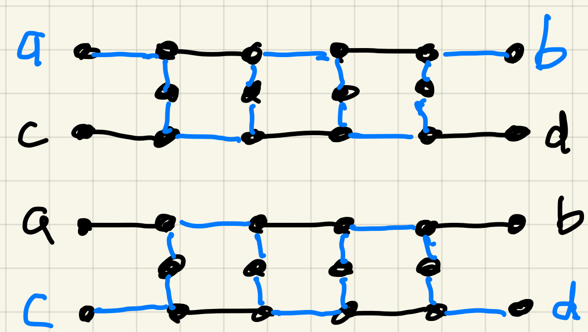
-

RHC with paired edges (RHCwpe) → RHC (3/3)

- Need to enhance G to enforce choices in C
- Gadget: $(e,e') = ((a,b),(c,d))$







Rudrata-Hamiltonian Cycle (RHC)

→ Traveling Salesperson Problem (TSP)

- RHC – find cycle visting each vertex once
- TSP – find shortest cycle visting each vertex once